RED SHIFT

- The red shift is most known cosmological phenomena.

$$\frac{\Delta \nu}{\nu} = \frac{\Delta \lambda}{\lambda} = \frac{v}{c} = z$$

v is velocity of an emitter which is moving from observer

$$\Delta \nu = \nu_e - \nu_o$$

$$\Delta \lambda = \lambda_o - \lambda_e$$

1. Red Shift

2. Deceleration parameter

3. Particle Horizon

4. Age of the Universe

The general description of the phenomena is as follows: a spectral line from another galaxy ia emitted with the same frequency as in laboratory. But observed frequency is different.

Let consider the motion of light rays in the expanding Universe. The light ray is moving along a straight line according to the equation ds=0. This equation is postulate of the Special Relativity Which is valid in general Relativity too. In the expanding Universe metric has form (flat hypersurface):

$$ds^2 = c^2 dt^2 - a^2(t)\left(dr^2 + r^2\left\{d\theta^2 + \sin^2\theta d\varphi^2\right\}\right)$$

Let assume that an observer is in the center of spherical coordinate system and the light rays move along the radial coordinate. So, we can put dθ=0, dϕ=0.

In this case metric equation is reduced to form

$$dt^2 - a^2(t)r^2 = 0$$ or

$$dt = \pm a(t)dr$$

One can solve this equation as

$$\int_{t_e}^{t_o} \frac{dt}{a(t)} = -r_o + r_e$$

In this equation t is physical time. One can introduce new variable η which is called conformal time. The equation for this time is .

$$d\eta = \frac{dt}{a(t)} = dr$$

In this case the solution of above equation is very simple

$$\eta_o - \eta_e = r_e - r_o$$

Now one can calculate the interval of an event in emitter and in observer. Suppose that the lagrangian distance (r) between the emitter and the observer is constant. In this case interval of an event at emitter position ($\Delta\eta$) is equal to interval at observer position:

$$\Delta\eta_e = \Delta\eta_o$$

One can rewrite this equation in terms of physical time as

$$\frac{dt_e}{a(t_e)} = \frac{dt_o}{a(t_o)}$$

Suppose that the event is one cycle of radiation. One can rewrite above equation in terms of frequency

$$\nu_o = \frac{a(t_e)}{a(t_o)}\nu_e \qquad \text{or}$$

$$\frac{a(t_o)}{a(t_e)} = 1 + z$$

as general definition of red shift

An example of redshift calculation

$$\eta_o - \eta_e = \frac{3}{a_o}\left[t_o - \sqrt[3]{t_e t_o^2}\right]$$

$$l_e = a_o r_e$$

$$H = \frac{1}{a(t)}\frac{da(t)}{dt} = \frac{2}{3}\frac{1}{t}$$

Now one can calculate the redshift of t_e epoch:

$$1+z=\left(\frac{t_o}{t_e}\right)^{2/3}$$

and relation of l_e as a function of redshift

$$\frac{H\ l_e}{2c}=1-\frac{1}{\sqrt{1+z}}$$

or

$$z=\frac{H\ l_e}{2c}\ \frac{\left(2-\frac{H\ l_e}{2c}\right)}{\left(1-\frac{H\ l_e}{2c}\right)^2}$$

for

$$\frac{H\ l_e}{2c} << 1$$

one obtain

$$z = \frac{H\ l_e}{c}$$

what is Hubble law, in the case $z \to \infty$

one obtain

$$\frac{H\ l_e}{2c} \to 1$$

To clarify the result let us consider the case of small z

$$z << 1$$

The general equation for redshift is

$$1 + z = \frac{a(\eta_o)}{a(\eta_e)} \quad \Rightarrow \quad z = \frac{a(\eta_o) - a(\eta_e)}{a(\eta_e)}$$

where $$\eta_e = \eta_o - r_e$$

Expand the scale factor into Taylor series

$$a(\eta_e) = a(\eta_o) - r_e \frac{da(\eta)}{d\eta}\Big|_{\eta=\eta_o} + \frac{1}{2} r_e^2 \frac{d^2 a(\eta)}{d\eta^2}\Big|_{\eta=\eta_o} - ....$$

Let us consider approximation to the first order of magnitude

$$l = a(\eta_o) r_e \qquad H = \frac{da(t)}{a(t)dt} = \frac{da(\eta)}{a^2(\eta)d\eta}$$

$$z = \frac{a_o - a_o + r_e \dot{a}_o}{a_o - r_e \dot{a}_o} \approx r_e \frac{\dot{a}_o}{a_o} = Hl$$

Let us consider approximation to the second order of magnitude

$$z = \frac{a_o - a_o + r_e\dot{a}_o - \frac{1}{2}r_e^2\ddot{a}_o}{a_o - r_e\dot{a}_o - \frac{1}{2}r_e^2\ddot{a}_o} \approx r_e\left(\frac{\dot{a}_o}{a_o} - \frac{r_e}{2}\frac{\ddot{a}_o}{a_o}\right)\left(1 + r_e\frac{\dot{a}_o}{a_o}\right)$$

$$z = Hl + \frac{1}{2}l^2\left(2H^2 - \frac{\ddot{a}_o}{a_o}\right)$$

Now introduce the definition. The deceleration parameter is:

$$q_o \equiv -\frac{\ddot{a}_o a_o}{\dot{a}_o^2}$$

In this case, the redshift is:

$$z = Hl + \frac{1+q_o}{2}(Hl)^2$$

Deceleration parameter depends both of geometry and type of matter in the Universe.

q=1/2 in the case of the dust dominated Universe

q=1 in the case of the radiation dominated Universe

THE CONCEPT OF PARTICLE HORIZON

The definition of cosmic distance to an object which emitted photon at t_e is:

$$l_e = a_o \int_{t_e}^{t_o} \frac{dt}{a(t)}$$

Suppose that we calculate the distance to the object which emitted photon at the moment of the Universe creation $t_e=0$
In this case l_p is called particle horizon

$$l_p = a_o \int_{0}^{t_o} \frac{dt}{a(t)}$$

This value is different in different models. For instance, in the dust dominated model of Universe

$l_p=3t_0=2H_0^{-1}$,

in the radiation dominated model of Universe

$l_p=2t_0=H_0^{-1}$.

So,

$$l_p \propto t$$

$a(t) \propto t^{2/3}, t^{1/2}$ physical distance between two particles is multiplication of lagrangian distance by scale factor

$$l_L \propto a(t)\xi_L \propto t^{2/3}, t^{1/2}\xi_L$$

It means that the horizon size is growing faster then separation between two test particles. Any particles will be inside horizon in the future. It means also that two test particles which are in opposite direction in space and which are coming into horizon just now have never been in contact in the past.

So, the size of particle horizon is the size of causally connected region of space in each moment.

One question arise.

Before I explain this question I would like to remind you some facts concerning the relic radiation. The relic radiation has almost the same temperature in different directions. But it practically does not interact with cosmic matter today and did not interact with it after the recombination. The hydrogen recombination temperature is $3000\ {}^oK$ which corresponds to $z=1000$ and $t_r = 10^{12}$ - 10^{13} sec. The size of causally connected regions on the sky at the moment of recombination was $2\ c\ t_r$. So, different parts of the sky with angular size

$$\theta = (1 + z_r)\left(\frac{t_r}{t_o}\right) \approx 1^0$$

should not "be aware" each other.

The temperature should be different and we could expect "spotness" sky with mean size of spot

$$\propto \quad 1^{0}$$

and significantly different temperature of spots

$$\approx 0.1 \quad {}^{0}K$$

We have not it. We could expect different properties of our Universe in different directions, but we have not it.

We have the homogeneous and isotropic Universe with accuracy of the order of 10^{-4}. It indicates that all observable region of our Universe was in causal contact in the past. This problem was one of metaphysical problem of classical Friedmannien cosmology and it was resolved by the inflation theory.

The definition of cosmic distance is:

$$r_e = a_o \int_{t_e}^{t_o} \frac{dt}{a(t)}$$

One can rewrite the definition for red shift in form:

$$\frac{a_o}{a(t)} = 1 + z(t)$$

and rewrite the cosmic distance to an object with red shift *z* in terms of red shift as

$$r_e = -\int_0^z (1+z)dt(z)$$

Also one can rewrite the equation for red shift in the form

$$a(t) = \frac{a_o}{1 + z(t)}$$

and obtain the Hubble parameter as function of red shift

$$H(t)dt = -\frac{dz}{1 + z}$$

and one can use this equation to calculate both distance to the object and the age of the Universe:

$$r_e = -\int_0^z \frac{dz}{H(z)}$$

$$\Delta t_e = -\int_0^z \frac{dz}{(1+z)H(z)}$$

It is time which pass from the moment t_e till present time.

and one can transform this equation into and obtain the equation which determines the age of the Universe:

$$t_0 = -\frac{c}{H_0}\int_0^\infty \frac{dz}{(1+z)\sqrt{\Omega_{mo}(1+z)^3 + \Omega_q + \Omega_{ro}(1+z)^4}}$$

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The End